



Nematic valley ordering in quantum Hall systems

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The interplay between quantum Hall ordering and spontaneously broken “internal” symmetries in two-dimensional electron systems with spin or pseudospin degrees of freedom gives rise to a variety of interesting phenomena, including novel phases, phase transitions, and topological excitations. Here we develop a theory of broken-symmetry quantum Hall states, applicable to a class of multivalley systems, where the symmetry at issue is a point-group element that combines a spatial rotation with a permutation of valley indices. The anisotropy of the dispersion relation, generally present in such systems, favors states where all electrons reside in one of the valleys. In a clean system, the valley “pseudospin” ordering occurs via a finite-temperature transition accompanied by a nematic pattern of spatial symmetry breaking. In weakly disordered systems, domains of pseudospin polarization are formed, which prevents macroscopic valley and nematic ordering; however, the resulting state still asymptotically exhibits the quantum Hall effect. We discuss the transport properties in the ordered and disordered regimes, and the relation of our results to recent experiments in AIAs.

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A remarkably diverse set of phases exhibiting the quantum Hall effect (QHE) are observed in sufficiently clean two-dimensional electron systems (2DESs) subjected to a high magnetic field.¹ Of these, a particularly interesting subset occurs in multicomponent QH systems, where in addition to the orbital degree of freedom within a Landau level (LL), electrons have low-energy “internal” degrees of freedom, such as spin or a “pseudospin” associated with a valley or layer index. QH states in such systems, in addition to the topological order, characteristic of all QH states, feature broken global spin/pseudospin symmetries—a phenomenon termed QH ferromagnetism (QHFM). The entangling of the charge and spin/pseudospin degrees of freedom leads to novel phenomena in QHFM states, including charged skyrmions,² finite-temperature phase transitions, and Josephson-type effects.^{1,3}

In the cases studied to date, the global symmetry is an internal symmetry that acts on spin/pseudospin. In this paper we study a situation where the global symmetry acts simultaneously on the internal index and on the spatial degrees of freedom. This occurs naturally in a multivalley system where different valleys are related by a discrete rotation so that valley (pseudospin) and rotational symmetries are intertwined. An example of such a system which is central to this paper is the AIAs heterostructure,^{4–6} where two valleys with ellipsoidal Fermi surfaces are present, as illustrated in Fig. 1(a).

This linking of pseudospin and space in this system has two significant consequences at appropriate filling factors such as $\nu=1$. First, in the clean limit the onset of pseudospin ferromagnetism, which occurs via a finite-temperature Ising transition, is necessarily accompanied by the breaking of a rotational symmetry that corresponds to nematic order, with attendant anisotropies in physical quantities. We shall call the resulting phase a quantum Hall Ising nematic (QHIN). Second, any spatial disorder, e.g., random potentials or strains, necessarily induces a random field acting on the pseudospins

which thus destroys the long-ranged nematic order in the thermodynamic limit. Interestingly, though, the resulting state *still* exhibits the QHE at weak disorder so we refer to it as the quantum Hall random-field paramagnet (QHRFPM). Although for concreteness we shall focus on the simple case of the $\nu=1$ state in AIAs heterostructures, our findings are readily extended to other values of ν and a variety of multivalley systems.

Symmetries. The only exact symmetries of QH systems are the discrete translational and point-group symmetries of the underlying crystalline heterostructures. However, in many circumstances, there are additional approximate symmetries, some of which are continuous. To the extent that spin-orbit coupling can be ignored, there is an approximate U(1) spin-rotation symmetry about the direction of the magnetic field. Since the magnetic length, $\ell_B = \sqrt{\hbar c / eB}$, and the Fermi wavelength, λ_F , are long compared to the lattice constant, the effective-mass approximation is always quite accurate, so it is possible to treat the translation symmetry as continuous. If the electrons occupy only a valley or valleys centered on the Γ point in the Brillouin zone, the effective-mass approximation also elevates a C_n point-group symmetry to a continuous U(1) rotational symmetry. Terms which

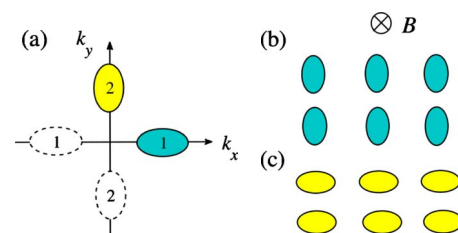


FIG. 1. (Color online) (a) Our model band structure. Ellipses represent lines of constant energy in the k space. There are two nonequivalent anisotropic valleys, 1 and 2. [(b) and (c)] Schematic representation of two types of order in the QHFM. The ellipses here represent LL orbitals in real space.

break this symmetry explicitly down to the discrete subgroup come from corrections to the effective-mass approximation, and so are smaller in proportion to $(a/\lambda_F)^2$, where a is the lattice constant of the semiconductor. All three of these approximate symmetries hold in GaAs heterostructures.

However, once there are multiple valleys centered on distinct symmetry-related Bloch wave vectors, the effective-mass tensor for each valley is, generically, anisotropic. Thus, already in the effective-mass approximation, individual valleys do not exhibit full rotational invariance; there are only the original discrete set of rotations which are associated with a simultaneous interchange of valleys. These discrete symmetries are unbroken for weak interactions in zero magnetic field. However, we show in this paper that in the presence of a strong magnetic field they are spontaneously broken at certain filling factors.

Specifically, in the two valley case considered explicitly here [Fig. 1(a)] the Hamiltonian has an approximate $Z_2 \times U(1)$ invariance: the Z_2 represents the operation of a $\pi/2$ rotation combined with valley interchange. The $U(1)$ reflects an approximate conservation of the valley index, which is violated only by the exponentially small Coulomb matrix elements, V_{iv} , which involve the intervalley scattering of a pair of electrons. The QHFM should thus exhibit a finite-temperature Z_2 or Ising symmetry-breaking phase transition, accompanied by a spontaneous breaking of the rotational symmetry from C_4 to C_2 , i.e., to Ising-nematic ordering. It is important to note that although V_{iv} breaks the approximate $U(1)$ symmetry, the Ising symmetry is exact.

We should note that there is well understood counter example to our general argument concerning the lack of continuous symmetries in multivalley systems which is realized at a (110) surface in Si. Here the 2DEG occupies two valleys centered at $\mathbf{k} = \pm \mathbf{Q}/2$, \mathbf{Q} being shorter than the smallest reciprocal-lattice vector. In this case, the only rotational symmetry is a symmetry under rotation by π . Yet, to the extent the intervalley scattering, V_{iv} , can be neglected, this problem was shown by Rasolt *et al.*⁷ to have an $SU(2)$ pseudospin symmetry. This derives from the fact that, in this case, the effective-mass tensors in the two valleys are *identical*. While in the case of interest to us there is only a discrete Z_2 symmetry, due to the effective-mass anisotropy in each valley, in the limit of small anisotropy there is a reference $SU(2)$ symmetry which is only weakly broken. For clarity, and without loss of generality, we will in places consider this analytically tractable limit, although in reality, the mass anisotropy in AAs is not small.

Ising anisotropy. The single-particle Hamiltonian in each of the valleys, labeled by the index $\kappa=1,2$, is given by $H_\kappa = \sum_{i=x,y} \frac{(p_i - K_{\kappa i} + eA_j/c)^2}{2m_{\kappa i}}$, where $\mathbf{K}_1=(K_0,0)$ and $\mathbf{K}_2=(0,K_0)$ are the positions of the two valleys in the Brillouin zone. We work in Landau gauge, $\mathbf{A}=(0,-Bx)$, in which eigenstates can be labeled by their momentum p_y that translates into the guiding center position $X=p_y \ell_B^2$. The lowest LL eigenfunctions in the two valleys are given by

$$\psi_{\kappa,X}(x,y) = \frac{e^{ip_y y}}{\sqrt{L_y \ell_B}} \left(\frac{u_\kappa}{\pi} \right)^{1/4} e^{-\frac{u_\kappa(x-X)^2}{2\ell_B^2}}, \quad (1)$$

where $\lambda^2 = (m_{1,x}/m_{1,y}) = (m_{2,y}/m_{2,x})$ is the mass anisotropy in terms of which $u_1 = 1/u_2 = \lambda$.

Proceeding to the effects of the Coulomb interactions, we notice that the terms in the Hamiltonian that involve intervalley-scattering processes require large momentum transfer, of order π/a , and therefore they are small in proportion to a/ℓ_B . In accord with that, we write the Hamiltonian as follows:

$$H = H_0 + H_{iv}, \quad H_0 = \frac{1}{2S} \sum_{\kappa,\kappa'} V(\mathbf{q}) \rho_{\kappa\kappa}(\mathbf{q}) \rho_{\kappa'\kappa'}(-\mathbf{q}), \quad (2)$$

where $S=L_x L_y$ is the system's area, $\rho_{\kappa\kappa}$ is the density component within valley κ , $V(\mathbf{q}) = \frac{2\pi e^2}{\epsilon q}$ is the matrix element of the Coulomb interaction, and H_{iv} denotes intervalley scattering terms,⁸ which we neglect for now.

To account for the spatial structure of LL wave functions, we follow the standard procedure of projecting the density operators onto the lowest LL (see, e.g., Ref. 3),

$$\rho_{\kappa\kappa}(\mathbf{q}) = F_{\kappa\kappa}(\mathbf{q}) \bar{\rho}_{\kappa\kappa}(\mathbf{q}), \quad F_{\kappa\kappa}(\mathbf{q}) = e^{-\left(\frac{q_x^2}{4u_\kappa} + u_\kappa \frac{q_y^2}{4} \right)}, \quad (3)$$

where the projected density operator is given by

$$\bar{\rho}_{\kappa\kappa}(\mathbf{q}) = \sum_{\bar{X}} e^{iq_x \bar{X}} c_{\kappa, X_+}^\dagger c_{\kappa, X_-}, \quad X_\pm = \bar{X} \pm \frac{q_y}{2}.$$

In the limit of vanishing mass anisotropy, $\lambda \rightarrow 1$, the Hamiltonian H_0 is $SU(2)$ symmetric, so at filling factor $\nu = 1$ there is a family of degenerate fully pseudospin polarized ground states, favored by the exchange interactions,

$$\Psi_{\alpha,\beta} = \prod_X (ac_{1,X}^\dagger + \beta c_{2,X}^\dagger) |0\rangle, \quad |\alpha|^2 + |\beta|^2 = 1. \quad (4)$$

In this notation, the components of the nematic order parameter are given by $n_x = \alpha\beta^* + \alpha^*\beta$, $n_y = i\alpha\beta^* - i\alpha^*\beta$, $n_z = |\alpha|^2 - |\beta|^2$, where $\mathbf{n}^2 = 1$. We can use the states [Eq. (4)] to obtain a variational estimate of the energy per electron of the system for different (uniform) values of the order parameter which should be reliable at least for λ near 1. The result is

$$E_0 = -\Delta_0(D_1 + D_2 n_z^2), \quad \Delta_0 = \frac{1}{2} \sqrt{\frac{\pi}{2}} \frac{e^2}{\epsilon \ell_B}, \quad (5)$$

where

$$D_1 = (C_1 + C_2)/2, \quad D_2 = (C_1 - C_2)/2, \quad (6)$$

$$C_1 = \frac{2K(\sqrt{1-1/\lambda^2})}{\pi \sqrt{\lambda}}, \quad C_2 = \sqrt{\frac{2\lambda}{1+\lambda^2}}, \quad (7)$$

K being the complete elliptic integral of the first kind. Clearly, when $\lambda \neq 1$, the $SU(2)$ symmetry is broken down to $Z_2 \times U(1)$ and the resulting QHIN indeed has an Ising (easy-axis) symmetry. The magnitude of the anisotropic part of the energy, $D_2 \Delta_0$, is pictured in Fig. 2. For the experimentally relevant case, $\lambda^2 \approx 5$ and $\kappa \approx 10$, the anisotropy reaches a relatively large value of 5 K at $B=10$ T. Let us also note, for subsequent use, that the Ising symmetry can be explicitly broken in experiments by the convenient application of a uniaxial strain,⁴ which then acts as a valley-Zeeman field.

Thermal properties. In order to understand the behavior of

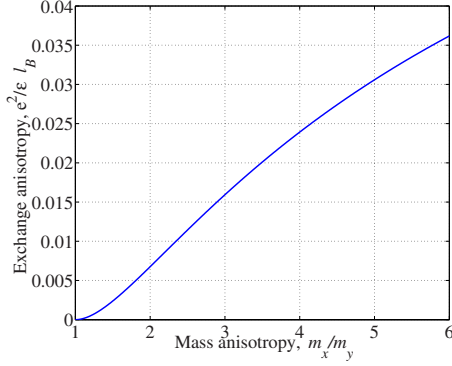


FIG. 2. (Color online) Easy-axis anisotropy of the QHFM as a function of underlying mass anisotropy.

the system more generally, in particular, to describe the properties of domain walls and excitations, we need to account for spatially varying order-parameter configurations. The classical energy functional for smooth configurations of the order parameter can be obtained approximately for $|\lambda - 1| \ll 1$ by the method of Ref. 3,

$$E[\mathbf{n}(\mathbf{r})] = \frac{\rho_s}{2} \int d^2r (\nabla \mathbf{n})^2 - \frac{\alpha}{2} \int d^2r n_z^2, \quad (8)$$

where $\alpha = D_2 \Delta_0 / 2\pi \ell_B^2$ and $\alpha \approx \frac{3}{32} \frac{\Delta_0}{2\pi \ell_B^2} (\lambda - 1)^2$. The symmetric part of the stiffness coefficient in Eq. (8) is given by, $\rho_s = \frac{e^2}{16 \cdot 2\pi \epsilon \ell_B} + O(\lambda - 1)$. In writing Eq. (8), we have neglected anisotropic stiffness terms of the form, $\frac{\rho_A}{2} \int d^2r n_z [(\partial_x \mathbf{n})^2 - (\partial_y \mathbf{n})^2]$ and $\frac{\rho_A'}{2} \int d^2r [3(\nabla n_z)^2 - (\nabla \mathbf{n})^2]$. While these terms are also quadratic in the gradient expansion, in the limit $|\lambda - 1| \ll 1$, the first term is at most cubic⁹ in $\lambda - 1$, such that $\rho_A \approx o[(\lambda - 1)^2]$, while the second term is quadratic, $\rho_A' = o[(\lambda - 1)]$, and so they are much smaller than the gradient term we have kept.

The nematic ordering temperature can readily be estimated from Eq. (8), which is precisely the continuum limit of the 2D Heisenberg ferromagnet with weak Ising anisotropy. Consequently, T_c vanishes for $\alpha = 0$, but only logarithmically, due to the exponential growth of correlations in the Heisenberg model,

$$k_B T_c \sim 4\pi \rho_s \log^{-1}[\rho_s / \alpha \ell_B^2]. \quad (9)$$

Since in reality the anisotropy is not too small, a more robust estimate is just $T_c \sim \rho_s$.¹⁰ This puts it in the range of several kelvin, well above typical temperatures at which quantum Hall experiments are carried out, which range from a few tens to a few hundred millikelvins.⁴

Domain walls and quasiparticles. The topological defects of an Ising ferromagnet are domain walls, in this case domain walls across which the valley-polarization changes sign. We obtain a domain-wall solution by minimizing the classical energy in Eq. (8) to obtain the length scale $L_0 = \sqrt{\frac{\rho_s}{\alpha}}$ which characterizes the domain-wall width and the surface tension, $\mathcal{J} \sim \sqrt{\rho_s \alpha}$, its creation energy per unit length. The domain-wall solution obtained in this way spontaneously breaks the approximate U(1) symmetry as the energy is

independent of the choice of the axis of rotation of \mathbf{n} in the plane perpendicular to n_z . Naturally, since the domain wall is a one-dimensional object, thermal or quantum fluctuations restore the symmetry, but at $T=0$, and in the absence of explicit symmetry-breaking perturbations, what remains is a gapless “almost Goldstone mode” and power-law correlations along the domain wall. A small gap in the spectrum and an exponential falloff of correlations beyond a distance ξ_{iv} are induced when the effects of the weak intervalley scattering terms, V_{iv} , are included.

In AIAs, the anisotropy is $\lambda^2 \approx 5$, so L_0 is only 30% greater than ℓ_B , which indicates that our treatment should be supplemented by microscopic calculations that can better handle a strong Ising anisotropy.⁹ A variational ansatz for a domain wall can be constructed of the same form as in Eq. (4) by treating α and β as (complex) functions of X , with asymptotic forms $(\alpha, \beta) \rightarrow (1, 0)$ as $X \rightarrow -\infty$ and $(\alpha, \beta) \rightarrow (0, 1)$ as $X \rightarrow \infty$. In the limit of large λ , the optimal such state consists of a discontinuous jump between these two limiting values across the domain wall so that the domain-wall width is simply equal to ℓ_B . When intervalley scattering is absent, such a wall, being a boundary between two different QH liquids (one with $\nu_1=0, \nu_2=1$ and the other with $\nu_1=1, \nu_2=0$), supports two counter-propagating chiral gapless modes—one with pseudospin “up” and the other with pseudospin “down.” Coulomb interactions between the two modes turn this into a type of Luttinger liquid. This connects smoothly to the description obtained above in the limit of weak anisotropy, and indeed the Luttinger liquid action can be derived explicitly from a σ -model description^{11,12} by augmenting the classical energy in Eq. (8) with an appropriate quantum dynamics.

The other excitations of interest are charged quasiparticles and it is well known that in the SU(2) limit at $\lambda=1$ they are pseudospin skyrmions of divergent size.² However, the smallness of L_0 at $\lambda^2 \approx 5$ alluded to above implies that for the experimentally relevant case the quasiparticles will be highly, if not completely, valley polarized.

Properties of the clean system. For $T < T_c$, where the pseudospin component n_z has a nonzero expectation value, C_4 rotation symmetry is spontaneously broken to C_2 . Thus, nonzero values of any nontrivial traceless symmetric tensor can also be used as an order parameter.

Ideally, thermodynamic quantities, for instance of the difference in the valley occupancies, provide the conceptually simplest measures of the broken symmetry. However, such quantities are not easily measured in practice. Following our remark above, we should just as well be able to use the experimentally accessible transport anisotropy ratio

$$N = \frac{\sigma_{xx} - \sigma_{yy}}{\sigma_{xx} + \sigma_{yy}} \neq 0 \quad (10)$$

as a measure of nematic order. However, at $T=0$, where $\sigma_{aa}=0$, N is ill defined. This problem can be resolved by either measuring σ_{aa} at finite temperature, $T > 0$, and then possibly taking the limit $T \rightarrow 0$. (Alternatively, one could imagine working at finite frequency, and then taking the limit as the frequency tends to 0.) However, in practice, the conductivity is strongly affected by the presence of even weak

disorder, so any practical discussion of the resistive anisotropy must be preceded by an analysis of the effects of disorder.

Length scales from weak disorder. By analogy with the random-field Ising model,¹³ we know¹⁴ that even an arbitrarily weak random valley-Zeeman field destroys the ordering of the QHIN, leading to formation of “Imry-Ma” domains of opposite valley polarization. In AAs such disorder can stem from random strains, which lead to position-dependent relative shifts of the energies of the two valleys. While the average strain (i.e., the average pseudomagnetic field) can be externally controlled,⁴ fluctuations of the strain are inevitable. Random fluctuations of the electric potential, V , also give rise to a random valley field.

The coupling of random strain and potential disorder to the QHIN order parameter is

$$E_{st} = \frac{1}{2} \int d^2r h(\mathbf{r}) n_z(\mathbf{r}), \quad (11)$$

where the random field $h(\mathbf{r}) = [h_{st}(\mathbf{r}) + h_{pot}(\mathbf{r})]$ with $h_{st}(\mathbf{r}) \propto \frac{\partial u(\mathbf{r})}{\partial x} - \frac{\partial u(\mathbf{r})}{\partial y}$, $u(\mathbf{r})$ being the displacement of point \mathbf{r} of the crystal, and

$$h_{pot}(\mathbf{r}) = \frac{(m_x - m_y) \ell_B^2}{2\pi\hbar^2} \left[\left(\frac{\partial V}{\partial x} \right)^2 - \left(\frac{\partial V}{\partial y} \right)^2 \right]. \quad (12)$$

On the basis of this analysis, we expect that the random valley-Zeeman field is smooth, with a typical correlation length $\ell_{dis} \gg \ell_B$. For weak disorder, the Imry-Ma domain size is set by the mean-squared strength of the random field (assumed to have zero mean and to be short-range correlated) $W \equiv \int d^2r \langle h(\mathbf{r}) h(\mathbf{0}) \rangle$ and the domain-wall energy per unit length, \mathcal{J} (defined above) as $\xi_{IM} \propto \exp[A(\mathcal{J})^2/W]$ where A is a number of order 1. Because of the exponential dependence on disorder, it is possible for ξ_{IM} to vary, depending on sample details, from microscopic to macroscopic length scales.

Disorder also leads to scattering between the valleys although this is again suppressed due to the mismatch between the reciprocal-lattice vector and the length scale of the dominant potential fluctuations. There is thus a second emergent length scale, ξ_{iv} , which is the length scale beyond which conservation of valley pseudospin density breaks down.¹⁵ However, this length scale approaches a finite value in the limit of vanishing disorder due to intervalley Coulomb scattering, discussed above. Different regimes of physics are possible depending on the ratio of ξ_{IM}/ξ_{iv} . Finally, especially when the filling factor deviates slightly from $\nu=1$, there is a length scale, ξ_{QP} , which characterizes the quasiparticle localization length. Because the magnetic field quenches the quasiparticle kinetic energy, even for extremely weak disorder, we expect $\xi_{QP} \sim \ell_B$ is relatively short.

Intrinsic resistive anisotropy. In a quantum Hall state at low temperatures, dissipative transport is usually due to hopping of quasiparticles between localized states, accompanied by energy transfer to other degrees of freedom.¹⁶ Typically, transport is of variable-range-hopping (VRH) type, such that the optimal hop is determined by the competition between energy offset of the two states and their overlap. We will now

apply these ideas to our system when its transport primarily involves hopping of electrons between localized states within one of the valleys. This requires (a) either that a uniaxial strain be applied to substantially eliminate domain walls and achieve valley polarization in the proximity of $\nu=1$ or that the sample be smaller than ξ_{IM} and (b) that ξ_{iv} be large compared to ξ_{QP} . For each valley the localization length is anisotropic, owing to the mass anisotropy, which results in the anisotropy of the corresponding contribution to the VRH conductivity.

The contribution to the resistive anisotropy from quasiparticles in valley 1, N_1 , can be computed as follows: first, we transform the anisotropic VRH problem into the isotropic one by rescaling coordinates, $x = \tilde{x}/\sqrt{\lambda}$ and $y = \sqrt{\lambda}\tilde{y}$. In the new coordinates the effective-mass tensor is isotropic, which, given the uncorrelated nature of the potential, implies that the VRH problem is isotropic,¹⁷ and therefore $\tilde{\sigma}_{xx} = \tilde{\sigma}_{yy}$. Since the ratio of the conductivities in the original coordinates is given by $\frac{\sigma_{xx}}{\sigma_{yy}} = \frac{1}{\lambda^2} \frac{\tilde{\sigma}_{xx}}{\tilde{\sigma}_{yy}}$,

$$N_1 = \frac{1 - \lambda^2}{1 + \lambda^2}, \quad (13)$$

which is negative for $\lambda > 1$, as expected, i.e., it is more difficult for particles to move in the direction of larger mass. Clearly, the resistive anisotropy produced by quasiparticles in valley 2 is $N_2 = -N_1$.

At $\nu=1$ localized states in both valleys are present, and due to combined particle-hole/valley-reversal symmetry of the state (in the absence of Landau-level mixing), the density of localized states should be same: the resistivity is thus expected to be isotropic. However, for $\nu \neq 1$, particle-hole symmetry is broken. Consider the case in which $n_z = +1$, which corresponds to filling valley 1 states. Then, at slightly different filling factor, $\nu = 1 - \delta\nu$ with $1 \gg \delta\nu > 0$, the density of localized states for valley $\kappa=1$ exceeds that for valley $\kappa=2$. Due to exponential sensitivity of the VRH conductivity to the density of states, this implies that the contribution of valley 1 to the total conductivity dominates, leading to an anisotropy of the total conductivity $N \approx N_1$. Conversely, for $\nu = 1 + \delta\nu$, $N \approx N_2 = -N_1$. It is worth noting that the scaling argument presented above for VRH regime is likely more general, and also applicable to the regime of thermally activated transport, which is relevant at intermediate temperatures.

Domain walls and the QHRFPM. We now move away from the above limit to where domain walls are a significant contributor to the transport—to systems much bigger than ξ_{IM} and at weak uniaxial strain. Now, dissipative transport is complicated by the existence of multiple emergent length scales. Transport within a nematic domain proceeds by variable-range hopping and/or thermal activation of quasiparticles. For length scales larger than ξ_{IM} , it is likely to be dominated by transport along domain walls, which will have insulating character or metallic character depending on whether viewed at distances large or small compared to ξ_{iv} .

A key question is whether the QHE survives the formation of domains. This is trickiest when no net valley-Zeeman field is applied where in the thermodynamic limit the domain

walls form a percolating network. In the limit $\xi_{iv} \rightarrow \infty$, the associated edge channels are conducting, and the domain-wall network can be expected to be well described by two copies of the Chalker-Coddington network model¹⁸ at criticality. This implies a critical metallic longitudinal dc conductivity of order e^2/h and the absence of QHE. However, at length scales longer than ξ_{iv} (or temperatures less than $T_{iv} \sim 1/\xi_{iv}$) the domain-wall states are localized, which implies a phase that exhibits the QHE without Ising/nematic order—the QHRFPM. Needless to say, in the absence of substantial amount of short-ranged disorder (which can produce a relatively small value of ξ_{iv}) the topological (quantum Hall) order in the QHRFPM is likely to be fragile. [In some ways similar results were obtained by Lee and co-workers,¹⁹ who considered an SU(2)-symmetric disordered QHFM, where magnetic order is destroyed by forming a spin glass without destroying the QHE.]

In the presence of a uniform valley-Zeeman field \bar{h} , which in the experiments on AAs can be controlled by applying uniform strain,⁴ the existence of the QHE is much more robust. Even weak fields can restore a substantial degree of valley density polarization as domains aligned with this field grow while those aligned opposite to it shrink. Consequently, the domain walls no longer percolate but rather are separated by a finite distance that grows with increasing \bar{h} . While we have yet to construct a detailed theory of the transport in this regime, it is clear that the characteristic energy scale characterizing the dissipative transport will rise rapidly from T_{iv} for $\bar{h}=0$, to the clean-limit gap $\Delta_0 \sim \rho_s$ for substantial values of \bar{h} . It is also important to note that this equilibrium response will come embedded in a matrix of dynamical phenomena characteristic of the random-field Ising model that can be translated straightforwardly to the case of the Ising nematic, as has been discussed in another context in Ref. 20. In particular, the macroscopic nematicity induced by the application of \bar{h} will be metastable for long times, even upon setting $\langle h \rangle = 0$ —thus giving rise to hysteresis.

Experiments. Turning briefly to experiments we note that an anomalously strong strain-induced enhancement of the apparent activation energy at $\nu=1$ has been observed^{4,21} in AAs, where it was tentatively attributed to the occurrence of valley skyrmions. As we noted earlier, in view of our estimate of a large Ising anisotropy skyrmions of the requisite size (about 15 flipped pseudospins²¹) are implausible. We would like to suggest that it is more plausible that these remarkable observations are associated with the growth of QHIN domains. In support of this idea, we have estimated the domain size from the long-ranged part of the potential

disorder alone and find that it should be order the distance to the dopant layer, $\xi_{dis} \approx 50$ nm which is thus much smaller than the system size. However, we currently lack a plausible estimate of ξ_{iv} which is sensitive to the short-ranged part of the disorder and which is needed to round out this explanation. Direct measurements of resistive anisotropies, and of hysteretic effects characteristic of the random-field Ising model²⁰ could directly confirm this proposal.

Related work. We note that there is a sizeable body of existing work on Ising QHFs produced at level crossings of different orbital LLs, which is typically achieved by applying tilted magnetic fields. These systems exhibit enhanced dissipation at coincidence^{22–24} which is the analog of a dissipation peak at zero valley-Zeeman field in our language. Qualitatively, our results concerning the domain-wall transport are consistent with this earlier work. Where we differ is in our contention that the domain walls do not, even at zero valley-Zeeman field, produce dissipation at $T=0$ —in the previous work^{25,26} this was not explicitly addressed in part as the focus was on accounting for the unexpected dissipation. The reader will also note that the QHIN studied here differs from the “nematic quantum Hall metal” (NQHM) phase which has been observed^{27,28} in ultraclean GaAs-GaAlAs heterostructures for fields at which the $n>1$ Landau level is nearly half filled. Unlike our system, the NQHM is a metallic state which does not exhibit the QHE but has a strongly anisotropic resistivity tensor.

In closing. The distinctive feature of our system is the breaking of a global symmetry that combines spatial and internal degrees of freedom. This physics and its attendant consequences will generalize immediately to other ferromagnetic fillings in the present system and then to other experimentally established examples of multivalley systems such as monolayer and bilayer graphene,^{29,30} where two valleys are present, and Si (111),³¹ where, depending on the parallel field, either four or six degenerate valleys can be present. Potentially, our ideas could apply farther afield in the case of three-dimensional Bi, where three electron pockets related by $2\pi/3$ and $4\pi/3$ rotations are present. Recently, high-field anomalies in transport and thermodynamic properties of Bi were found,^{32,33} which may indicate spontaneous breaking of the Z_3 valley symmetry driven by magnetic field, reminiscent of QHFM.

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